Covid-19 Epidemic Model with Constraints and Feedback

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"All models are wrong, but some are useful."
George Box

April 27, 2020

Problem Statement

What?

- Build a simple epidemic model able to capture the lockdown-relaxation, and possible re-infection (due to virus mutation for example).

Why?

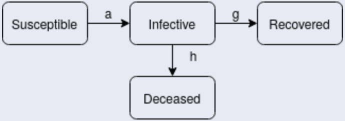
- Relaxing the lockdown constraints could flare up the epidemic.
- Possible re-infection feedback may perpetuate the epidemic for a long time.

How?

- SIRD population model: Susceptible, Infective, Recovered, Deceased.
- Constraints: lockdown-relaxation.
- Feedback: re-infection.

SIRD vanilla model

- Standard differential equations model, currently used by everybody:



$$\frac{dS}{dt} = -aSI$$
$$\frac{dI}{dt} = aSI - gI - hI$$
$$\frac{dR}{dt} = gI$$
$$\frac{dD}{dt} = hI$$

- Cannot capture lockdown-relaxation!
- Cannot capture re-infection!

SIRD Scaling Properties

- Scaling doesn't change the shape of the solution.
- Assume that this is the SIRD model before scaling:

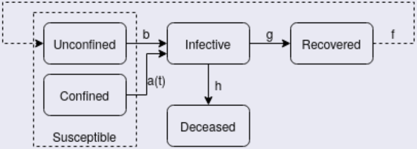
$$\frac{dS}{dt} = -\bar{a}SI$$
$$\frac{dI}{dt} = \bar{a}SI - \bar{g}I - \bar{h}I$$
$$\frac{dR}{dt} = \bar{g}I$$
$$\frac{dD}{dt} = \bar{h}I$$

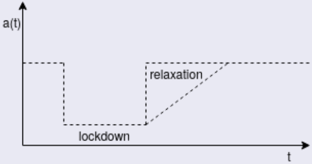
- One can conveniently scale the model such that:

$$g + h = 1$$
$$g = \bar{g}/(\bar{g} + \bar{h})$$
$$h = \bar{h}/(\bar{g} + \bar{h})$$
$$a = \bar{a}/(\bar{g} + \bar{h})$$

- That is, we divide the system parameters by $\bar{g} + \bar{h}$.

SIRD with constraints and feedback





$$\frac{dC}{dt} = -aCI \longleftarrow \text{Confined, 80\% (models lockdown)}$$

$$\frac{dU}{dt} = -bUI + fR \longleftarrow \text{Unconfined, 20\% (models lockdown)}$$

$$\frac{dI}{dt} = aCI + bUI - gI - hI$$

$$\frac{dR}{dt} = gI - fR$$

$$\frac{dD}{dt} = hI$$

$$a = a(t) \longleftarrow \text{(models lockdown-relaxation)}$$

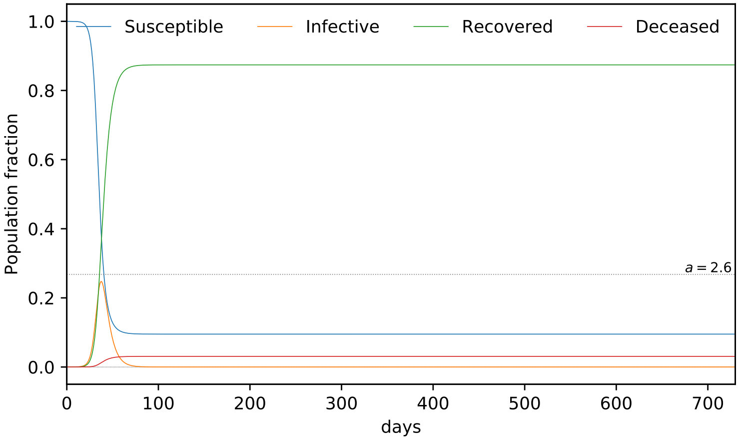
$$f > 0 \longleftarrow \text{(models re-infection)}$$

The Model: Lockdown-Relaxation and Possible Re-infection

Main Assumptions

- Susceptible = Confined (80%) + Unconfined (20%)
- Initial fraction of infective: $I(0) = 10^{-5}$
- Integration step: $\delta = 10^{-3}$
- Time scaling: $1000\delta = 5$ days (an infected can be infective for 5 days).
- Reproduction number for Confined: $a = 1$. This is the number of infections resulting form a single infection in the Confined population.
- Reproduction number for Unconfined: $b = 2.6$. This is the number of infections resulting form a single infection in the Unconfined population.
- Rate of Death: $h = 0.034$
- Rate of Recovered: $g = 1 - h$
- Re-infection feedback rate: $f = 0.05$
- Lockdown starts after: 20 days
- Lockdown duration: 40 days
- Relaxation time: variable from 0 days to 80 days
- Simulation time: 730 days (2 years)

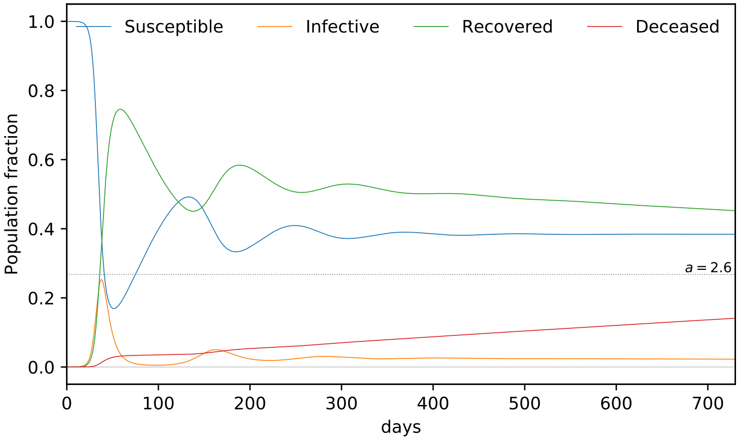
SIRD - vanilla model



SIRD - pure model

- $a = b = 2.6,\ f = 0,\ g = 0.966,\ h = 0.034$
- This is what would have happened without any lockdown measures and without re-infection feedback. Short and painful effect.

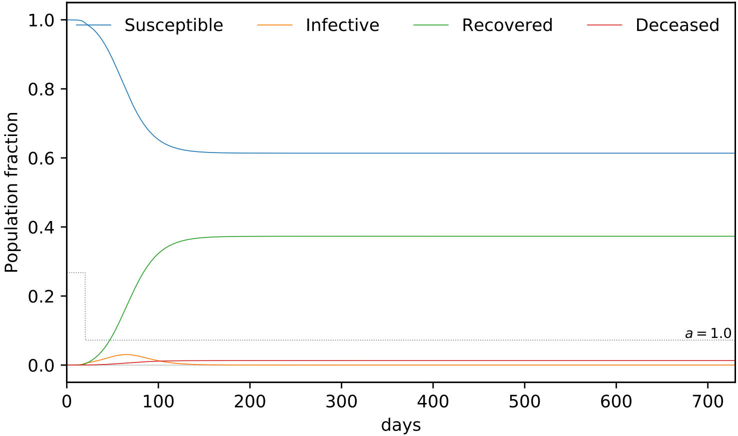
SIRD with re-infection feedback



SIRD with re-infection feedback

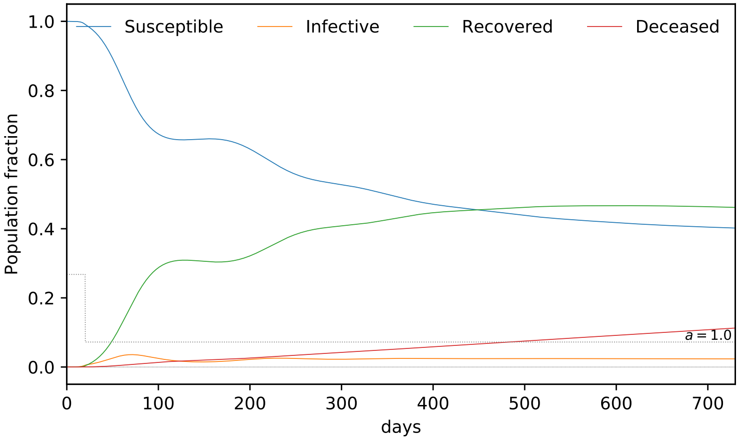
- $a = b = 2.6$, $f = 0.05$, $g = 0.966$, $h = 0.034$
- No lockdown, but 5% of Recovered become re-infected. Waves of epidemics, number of deaths increases substantially.

SIRD with perfect lockdown



SIRD with perfect lockdown

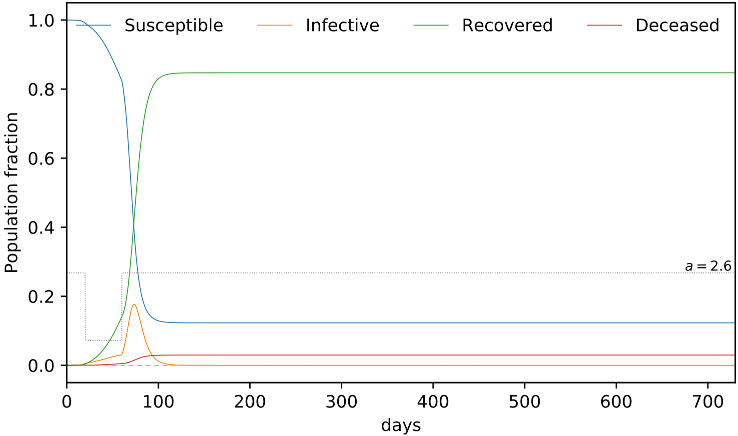
- $a = 1$ if $t \geq 20$, $a = b$ if $t < 20$, $b = 2.6$, $f = 0$, $g = 0.966$, $h = 0.034$
- 80% of population in continuous lockdown after 20 days from the onset of epidemic. Small number of deaths, no epidemic waves.



SIRD with perfect lockdown and re-infection feedback

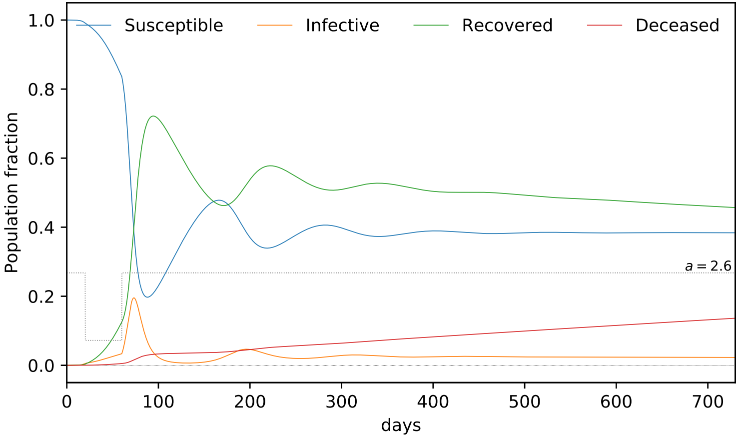
- $a = 1$ if $t \geq 20$, $a = b$ if $t < 20$, $b = 2.6$, $f = 0.05$, $g = 0.966$, $h = 0.034$
- 80% of population in continuous lockdown after 20 days, 5% feedback. Smaller number of deaths and epidemic waves.

SIRD with step lockdown-relaxation



SIRD with step lockdown-relaxation

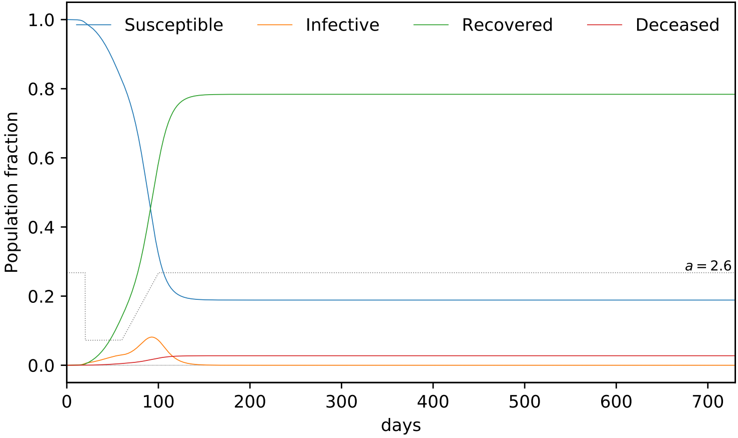
- $a = 1$ if $20 \leq t < 60$, $a = b$ otherwise, $b = 2.6$, $f = 0$, $g = 0.966$, $h = 0.034$. 80% of population in lockdown for 40 days, no re-infections. Big infective peak after relaxing the lockdown.



SIRD with step lockdown-relaxation and re-infection feedback

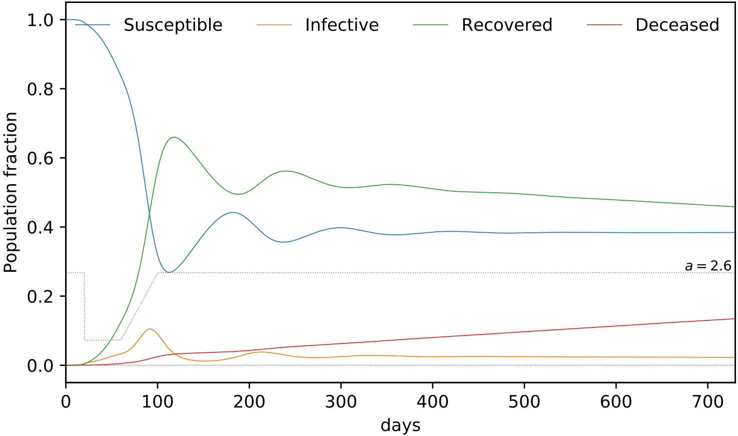
- $a = 1$ if $20 \leq t < 60$, $a = b$ otherwise, $b = 2.6$, $f = 0.05$, $g = 0.966$, $h = 0.034$. 80% of population in lockdown for 40 days, 5% feedback. Big infective peak after relaxing the lockdown.

SIRD with step-linear lockdown-relaxation



SIRD with step-linear lockdown-relaxation

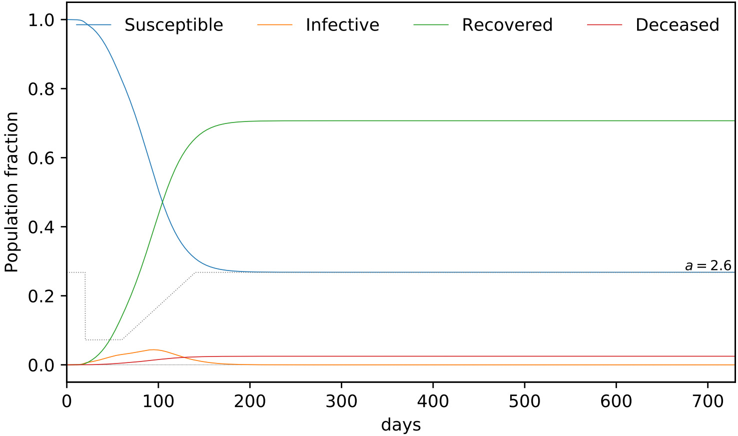
- $a = 1$ if $20 \leq t < 60$, $a = a(t), 60 \leq t < 100$, $b = 2.6$, $f = 0$, $g = 0.966$, $h = 0.034$. 80% of population in lockdown for 40 days, no re-infections. Linear relaxation 40 days. Medium infection peak during the relaxation.



SIRD with step-linear lockdown-relaxation and re-infection feedback

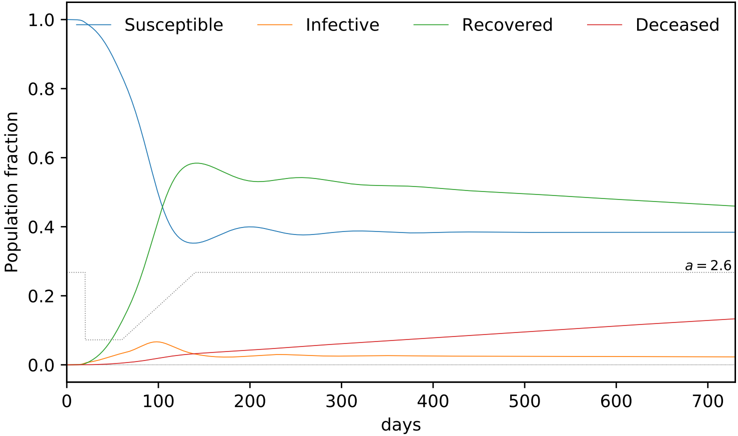
- $a = 1$ if $20 \leq t < 60$, $a = a(t), 60 \leq t < 100$, $b = 2.6$, $f = 0.05$, $g = 0.966$, $h = 0.034$. 80% of population in lockdown for 40 days, 5% feedback. Linear relaxation 40 days. Medium infection peak and waves.

SIRD with step-linear lockdown-relaxation



SIRD with step-linear lockdown-relaxation

- $a = 1$ if $20 \leq t < 60$, $a = a(t), 60 \leq t < 140$, $b = 2.6$, $f = 0$, $g = 0.966$, $h = 0.034$. 80% of population in lockdown for 40 days, no re-infections. Linear relaxation for 80 days. Smaller infection peak during the relaxation.



SIRD with step-linear lockdown-relaxation and re-infection feedback

- $a = 1$ if $20 \leq t < 60$, $a = a(t), 60 \leq t < 140$, $b = 2.6$, $f = 0.05$, $g = 0.966$, $h = 0.034$. 80% of population in lockdown for 40 days, 5% feedback. Linear relaxation for 80 days. Smaller infection peak and waves.

Conclusions

- The step lockdown-relaxation cannot flatten the peak of possible epidemic flare up.
- The step-linear lockdown-relaxation requires long time to flatten the peak of possible epidemic flare up.
- The possibility of a re-infection feedback is particularly worrying because it can create waves of epidemics and it can keep the virus active for long time.
- The number of deaths can be very high if the death rate is 3.4% (current WHO estimate), and there is re-infection feedback.

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Thank You!